Columbia Engineering
The Fu Foundation School of Engineering and Applied Science

ELEN E3106/4106 Lecture 6

Diffusion of Carriers

Outline

- Drift current (loose ends)
- Diffusion processes
- Diffusion & drift
- Built-in fields
- Diffusion with recombination
- Continuity equation & diffusion length

Assignments:
Reading: Streetman and Banerjee §4.4.1-4.4.4
Homework 2 due Friday Sept 19th by 5pm

Back to Basics: Relationship between Drift Current and Resistance

- Recall from Lecture 6,

$$\boxed{R = \frac{\rho L}{wt} = \frac{L}{wt}\frac{1}{\sigma}} \quad \boxed{\sigma = qn\mu_n + qp\mu_p} \quad \boxed{v_n = -\mu_n E}$$

- Where usually only _____________ carrier component dominates conductivity

- How can we back out current? Let's imagine we have a n-type material:

Ohm's law: _________ $\rightarrow I = \frac{V}{R} = \frac{V(wt\sigma)}{L} = \frac{VA(qn\mu_n)}{L} = qAn(\mu_n E) = qAnv_n$

Recap of carrier motion/currents (so far!)

- Recall from Lecture 4, we learned that there is random motion (_______________) of e^- and h^+ in the absence of E-field
  - Net motion of carriers = ____
  - E-field = 0, but T > 0, carriers move with thermal velocity,

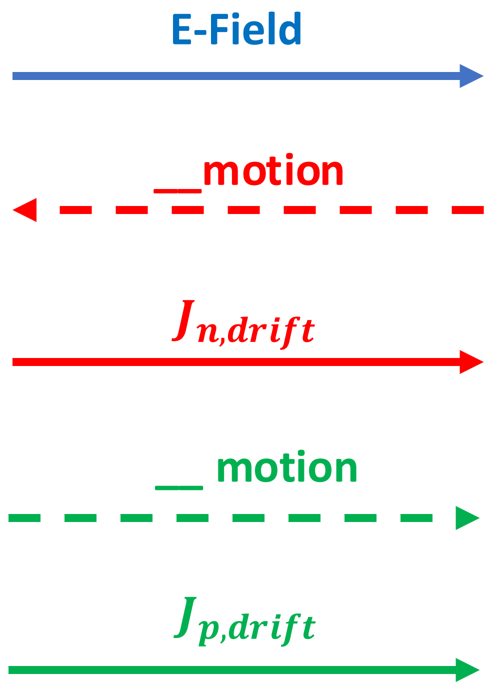
$$v_{th} = \sqrt{\frac{3kT}{m^*}}$$

- In the presence of an E-field, carriers will have **drift** velocity, $v_d = \pm$______
- Where the carrier mobility (ease with which carriers move in semi) is,

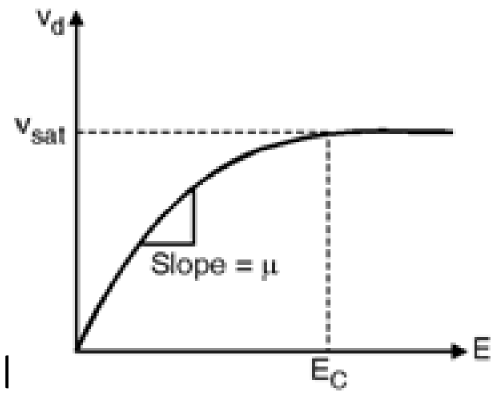
$$\_ = -\frac{q\tau_C}{m^*}$$

- So the net current in the presence of an E-field is,

$$J_{drift} = J_{n,drift} + J_{p,drift}$$



High Field Effects on Drift Velocity



Velocity saturation

- Recall from last lecture,

$$\boxed{J_n^{drift} = -qnv_{dn} = qn\mu_n E} \quad \boxed{J_p^{drift} = +qpv_{dp} = qp\mu_p E}$$

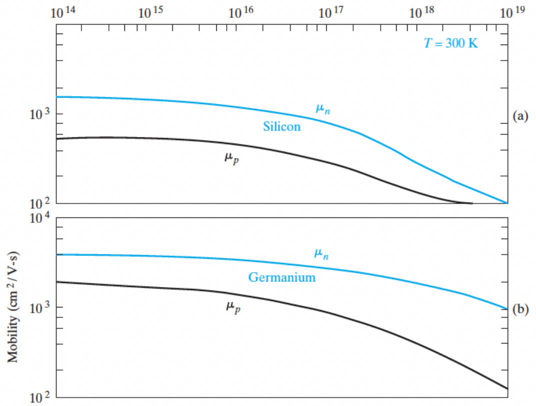
- Which '=' sign do we want?
  - The first '=' sign ________________, and we can find the drift curr
    - $J_{n,drift} = -qnv_{dn}$
    - $J_{p,drift} = qpv_{dp}$
- At ____________________, Ohm's law is valid (current density is directly proportional to electric field), and the second '=' sign applies:
  - We call this the ________________,
    - $J_{n,drift} = qn\mu_n E$
    - $J_{p,drift} = qp\mu_p E$
- At ______________________, (≥ ________________), electrons will reach **drift** ________________ and exhibit a sublinear dependence on the electric field (e.g. we use first '=' sign because $v_{d,sat} \neq \mu E$)

Sources: Electronics-Tutorial.net

Problem: Drift Current Calculations

- A 2 cm long piece of Si with cross-sectional area of 0.1 cm^2 is doped with donors at 10^{15} cm^{-3}, and has a resistance of 90 Ω. The saturation velocity of electrons in Si is 10^7 cm/s for fields above 10^5 V/cm. Calculate the electron drift velocity, if we apply a voltage of 100 V across the piece. What is the current through the piece if we apply a voltage of 10^6 V across it?

- First, we need to find the electric field,
- $E = \frac{across\ semiconductor}{of\ semiconductor} = \frac{100\ V}{2\ cm} = \_\_\_\_\_\ V/cm$ --> Which regime are we in? ____________



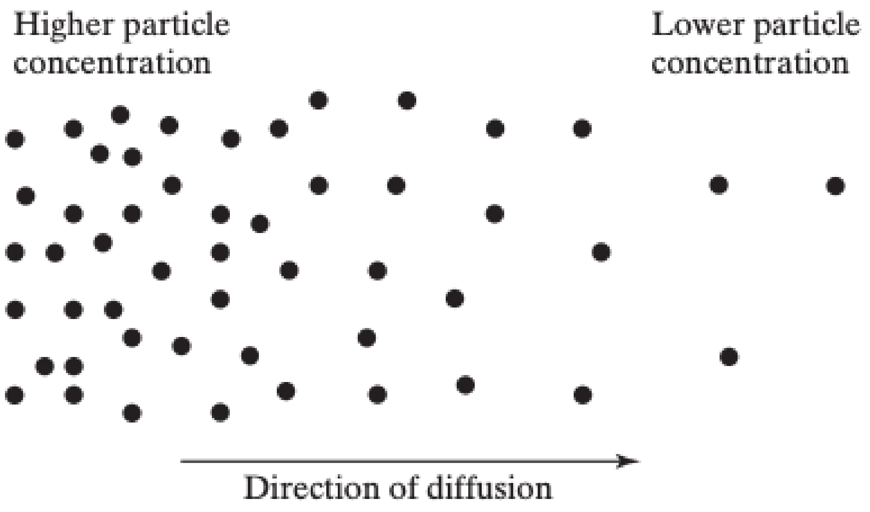
- We can estimate the mobility from Figure 3-23 and solve,
- $v_d = \mu_n E = \left(\_\_\_\_\_\_\_\_\frac{cm^2}{V \cdot s}\right)\left(50\frac{V}{cm}\right) =$ 7500 cm^2/s
- Now for the larger electric field,
- $E = \frac{10^6\ V}{2\ cm} = 5 \times 10^5\ V/cm$ --> ____________
- Using current equation from slide 2,
- $I = \_\_\_\_\_\_\_\_ \quad = (1.6 \times 10^{-19} C)(0.1\ cm^2)(10^{15}\ cm^{-3})\left(10^7\frac{cm}{s}\right) = 100\ A$

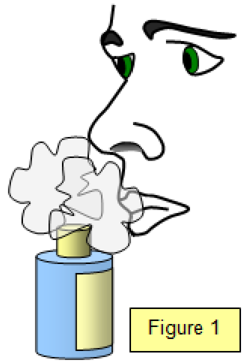
Figure 3-23 in textbook. Use for extrinsic semiconductors. Note you should use the total # impurities (Na+Nd)

Sources: Textbook

Diffusion

- In addition to ______, there is a second component to current called ________
- Particles move from point of higher concentration to a point of lower concentration
- What are some other real world examples of diffusion processes?
    - ____
    - ____



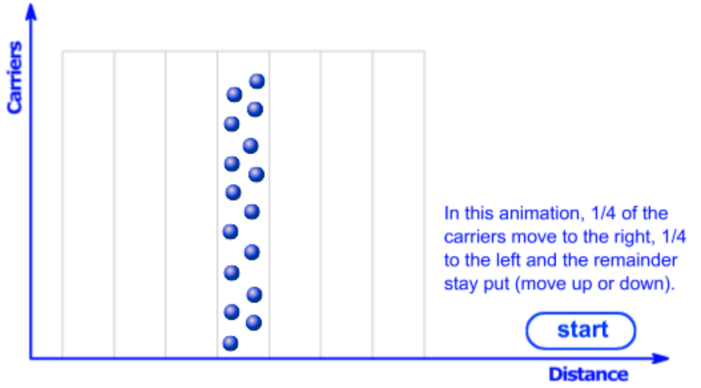


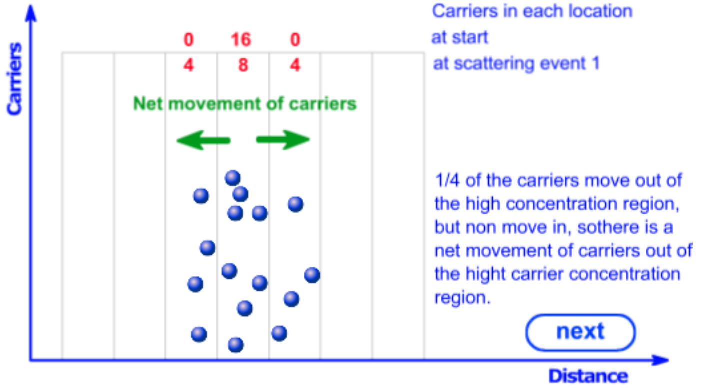
Diffusion

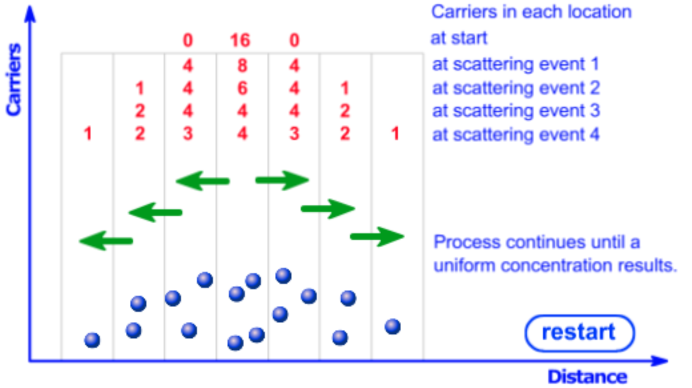
Sources: C. Hu, *Modern Semiconductor Devices for Integrated Circuits,* School Physics, Science Prof Online

What drives diffusion processes?

- Carriers undergo ________________and collide with each other, and ________
- They are moving in ______________until they collide, and move in new directions
- Carriers will have net thermal motion from high to low concentration areas
- When does diffusion stop? Once there is no longer a concentration gradient (e.g. the concentration is ______________)







Sources: https://www.pveducation.org/pvcdrom/pn-junctions/diffusion#:~:text=Diffusion%20is%20the%20random%20scattering,in%20cm2s%2D1.

Diffusion Current

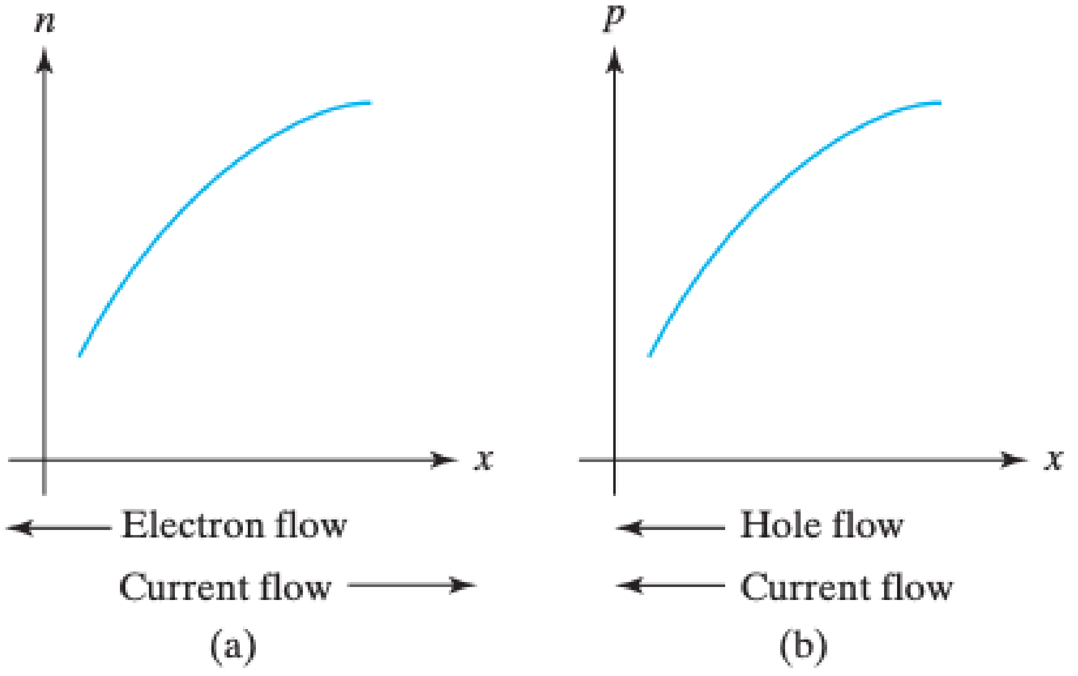
- What drives the net diffusion current?

- Will we have diffusion current in a uniform sample?

- The rate of diffusion is proportional to the concentration gradient,

$$J_{n,\text{diffusion}} \propto \frac{dn}{dx}$$

- And for holes?



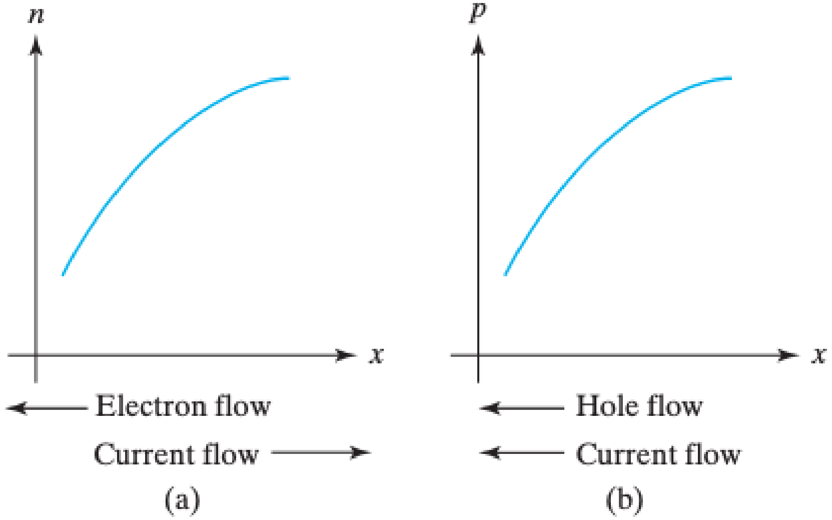
Sources: C. Hu, *Modern Semiconductor Devices for Integrated Circuits*

Diffusion Current Equations and Coefficients

- Mathematically, we can write the diffusion current densities,

$$\boxed{J_{n,\text{diffusion}} = qD_n\frac{dn}{dx}}$$

$$\boxed{J_{p,\text{diffusion}} = -qD_p\frac{dp}{dx}}$$



- D_n and D_p are the ____________________
  - Units?
- Why the (-) sign? The net motion of e- due to diffusion is in the direction of _______________ e⁻ concentration, so derivative is (-)dn/dx. (-q)(-dn/dx) --> we get (+)
- Are the diffusion currents in the same direction?

Sources: C. Hu, *Modern Semiconductor Devices for Integrated Circuits*

Total Currents and Visualizing Particle Motion

- In general in semiconductors, there can be ___ possible current sources:
    1. Electron drift
    2. Electron diffusion
    3. Hole drift
    4. Hole diffusion

- For electrons:

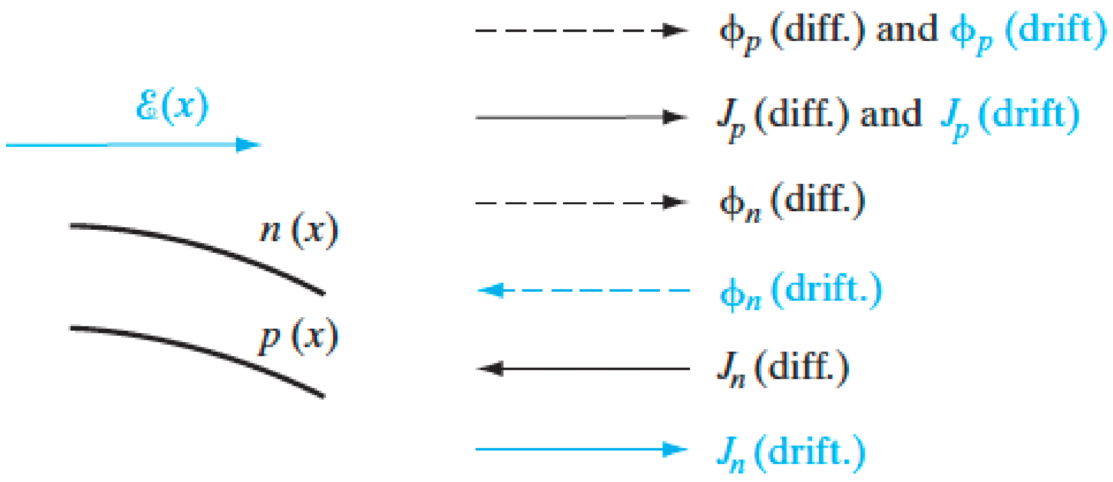
$$\boxed{J_n = J_{n,\text{drift}} + J_{n,\text{diffusion}} = qn\mu_n\mathscr{E} + qD_n\frac{dn}{dx}}$$

- For holes:

$$\boxed{J_p = J_{p,\text{drift}} + J_{p,\text{diffusion}} = qp\mu_p\mathscr{E} - qD_p\frac{dp}{dx}}$$

- And we can write the total current as the sum,

$$J =$$



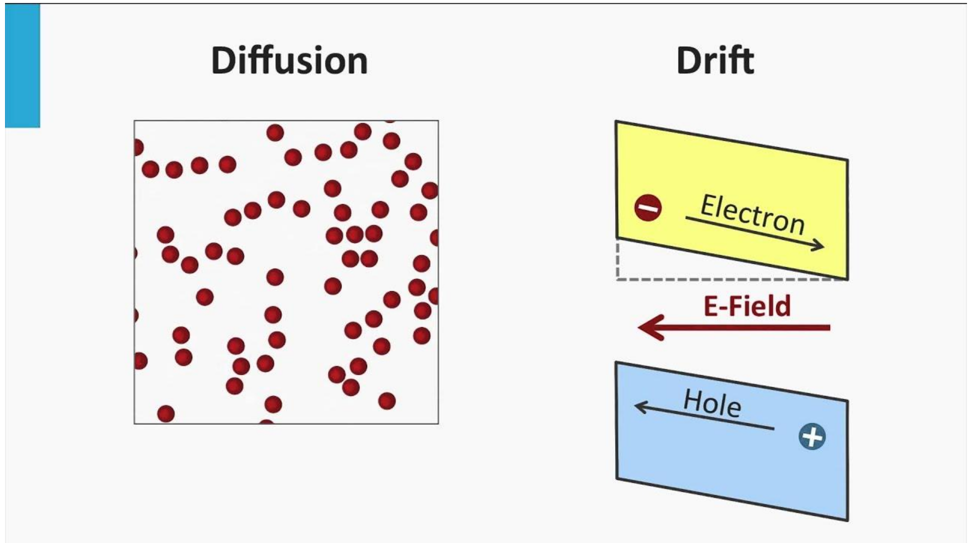
Dashed lines (___) denote direction of particle motion. Solid lines denote the resulting current direction

Sources: C. Hu, *Modern Semiconductor Devices for Integrated Circuits*

The Influence of Majority Carriers on Diffusion

- Important: __________ carriers rarely contribute much to drift current (there are too few of them),

$$J_n = J_{n,\mathrm{drift}} + J_{n,\mathrm{diffusion}} = qn\mu_n \mathscr{E} + qD_n \frac{\mathrm{d}n}{\mathrm{d}x}$$



- BUT if their gradient is high enough….

- Result: minority carrier diffusion currents can sometimes be _______ as majority carrier currents

Sources: E. Pop ECE 340 Slides, A. Smets TU Delft Lecture Week 2

Built-In Fields

- Under equilibrium, open-circuit conditions, the total current must always be ______
- No net current flows (I.e. J_{drift} = - J________)

- So any disturbance (e.g. light, doping gradient, thermal gradient) which may set up a carrier concentration gradient will also internally set up a built-in ____________

- What does this tell us? There must be some relationship between diffusion and drift. We can set the earlier total current equation equal to zero,

$$J_n = J_{n,\mathrm{drift}} + J_{n,\mathrm{diffusion}} = qn\mu_n\mathscr{E} + qD_n\frac{\mathrm{d}n}{\mathrm{d}x}$$

Sources: E. Pop ECE 340 Slides, A. Smets TU Delft Lecture Week 2

The Einstein Relation between D and μ

- Solving the equation (noting the equilibrium Fermi level does not vary with x, and the derivative of ____ as given in the textbook Eq. 4-26), we get:

$$\boxed{\frac{D}{\mu} = \frac{kT}{q}}$$

- This is called the ____________________ and is valid for ________________
- What does this allow us to calculate?

- This relation (almost) always holds true
- Physically, all scattering mechanisms (e.g. phonon/lattice and impurity scattering) that impede carrier _____ also impede carrier ______!

Table 4–1 Diffusion coefficient and mobility of electrons and holes for intrinsic semiconductors at 300 K. *Note:* Use Fig. 3–23 for doped semiconductors.

	D_n (cm^2/s)	D_p (cm^2/s)	μ_n (cm^2/V-s)	μ_p (cm^2/V-s)
Ge	100	50	3900	1900
Si	35	12.5	1350	480
GaAs	220	10	8500	400

Sources: Textbook

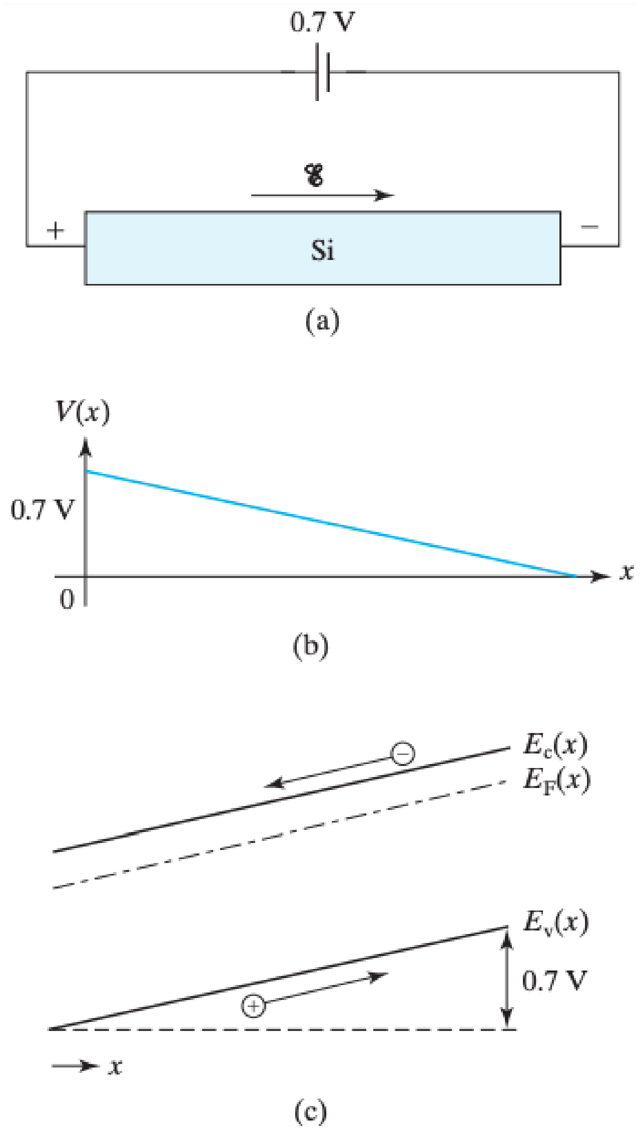
Relationship between Energy Diagrams, Voltage, and E-field

- When a voltage is applied across a piece of semiconductor, it ____________________
- (__) V raises the potential energy of a (+) charge and lowers the P.E. of a (-) charge

$$\mathscr{E}(x) = -\frac{d\mathscr{V}(x)}{dx}$$

- Therefore, a (+) V __________ the energy diagrams since we are plotting the energy of an _______________
- How do we convert from V to eV? Multiply by *q*

$$E_c(x) = \text{constant} - qV(x)$$



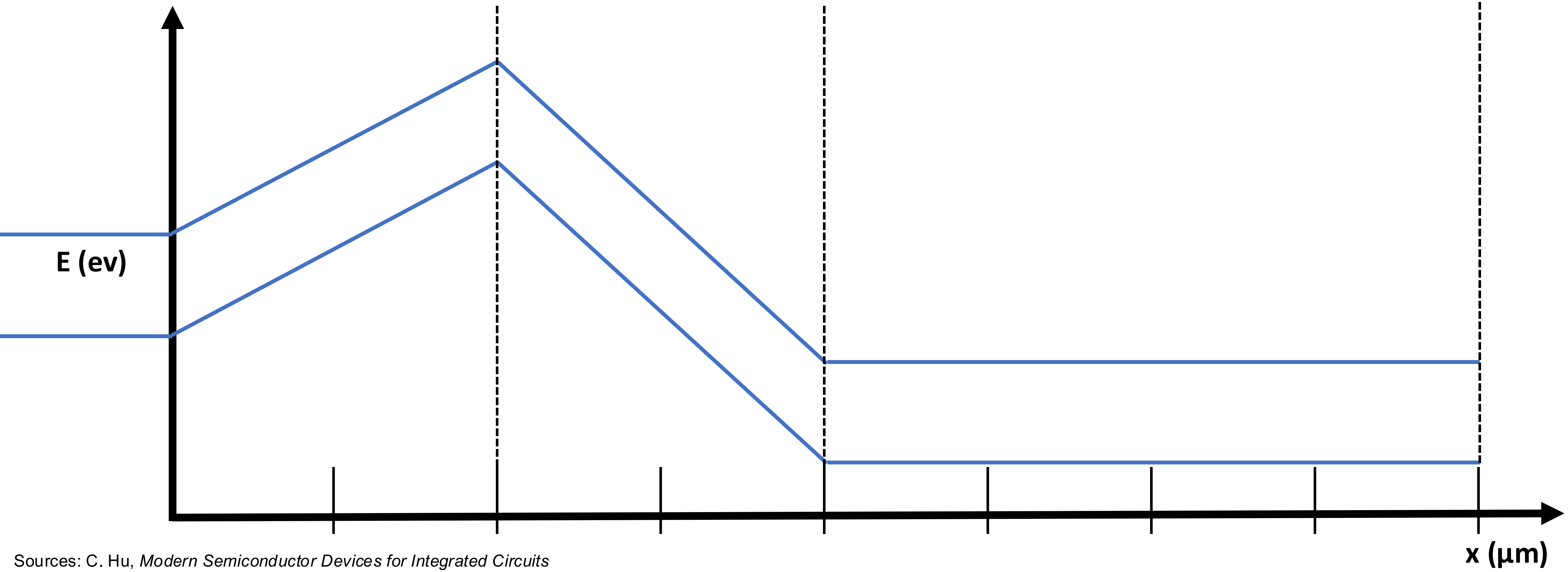
Energy band diagram of a semiconductor under applied voltage (+7V).

Sources: Textbook

Problem: Sketching Energy Band Diagrams with Applied Voltage

A semiconductor has a bandgap of 1 eV. It is subjected to the following potentials at the various locations as follows (assume linear variation of potentials between locations):

- Point A at x = 0 μm, V = 0 V
- Point B at x = 2 μm, V = -2 V
- Point C at x = 4 μm, V = +4 V
- Point D at x = 8 μm, E-field is zero between C and D



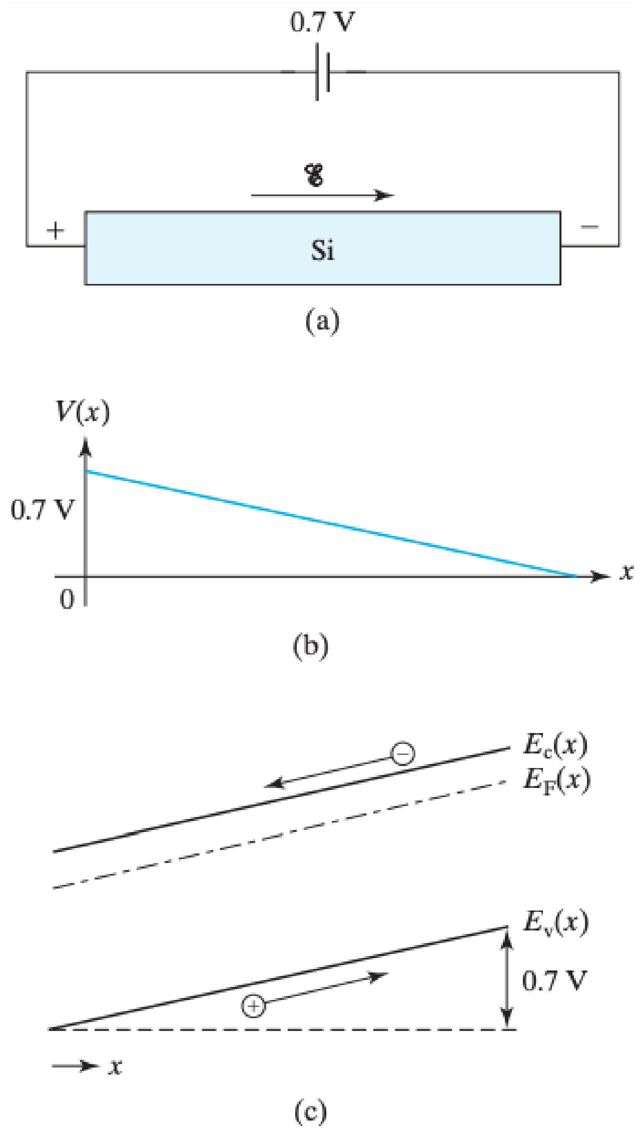
Sources: C. Hu, *Modern Semiconductor Devices for Integrated Circuits*

Relationship between Energy Diagrams, Voltage, and E-field

- Practical points to note:
  - E_c and E_v are higher where the voltage is ______
  - The slope of E_c and E_v indicates the E-field

$$\mathscr{E}(x) \equiv -\frac{\mathrm{d}V}{\mathrm{d}x} = \frac{1}{q}\frac{\mathrm{d}E_c}{\mathrm{d}x} = \frac{1}{q}\frac{\mathrm{d}E_v}{\mathrm{d}x}$$

- Useful analogies:
  - The e- will _________________ in the energy band diagram
  - The h+ will ______________
- Recall: both are seeking lowest energy state (______ of bands)



Energy band diagram of a semiconductor under applied voltage (+7V).

Sources: Textbook

Looking at Built-In Fields Again

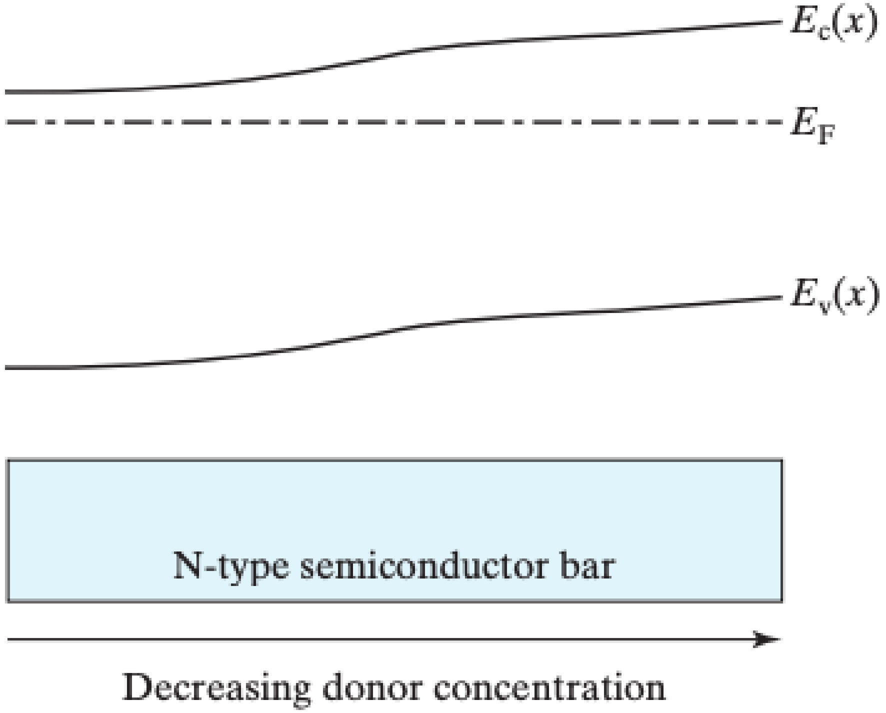
- Now that we understand the effects of voltage and fields on the band diagram, we can perhaps better understand the built-in fields that arise from concentration gradients in equilibrium

$$\mathscr{E}(x) \equiv -\frac{\mathrm{d}V}{\mathrm{d}x} = \frac{1}{q}\frac{\mathrm{d}E_c}{\mathrm{d}x}$$

- In equilibrium: Fermi level is _________

- Left side more _________ doped than right: E_c is closer to E_F

- Because E_c is not __________, an E-field will be created as real as a field created by an ______________ voltage



Sources: Textbook

Problem: Calculating carrier diffusion

The hole density in an n-type silicon wafer ($N_D = 10^{17}$ cm^{-3}) decreases linearly from 10^{14} cm^{-3} to 10^{13} cm^{-3} between x = 0 and x = 1 µm. Calculate the hole diffusion current.

- Rearranging the Einstein relation, and using the plot to estimate mobility,
  - $\underline{\hspace{4cm}} = 0.026(317) = 8.2\,\frac{cm^2}{s}$
- What is dp/dx? Change in carrier concentration over change in distance:
  - $\frac{dp}{dx} = \frac{\hspace{2cm}}{10^{-4}\ cm}$
- Now we can use the diffusion current equation,
  - $J_{p,diffusion} = qD_p\frac{dp}{dx} = (1.62 \times 10^{-19}C)\left(8.2\,\frac{cm^2}{s}\right)\left(\frac{9\times10^{13}cm^{-3}}{10^{-4}cm}\right) =$ $1.18\ A/cm^2$

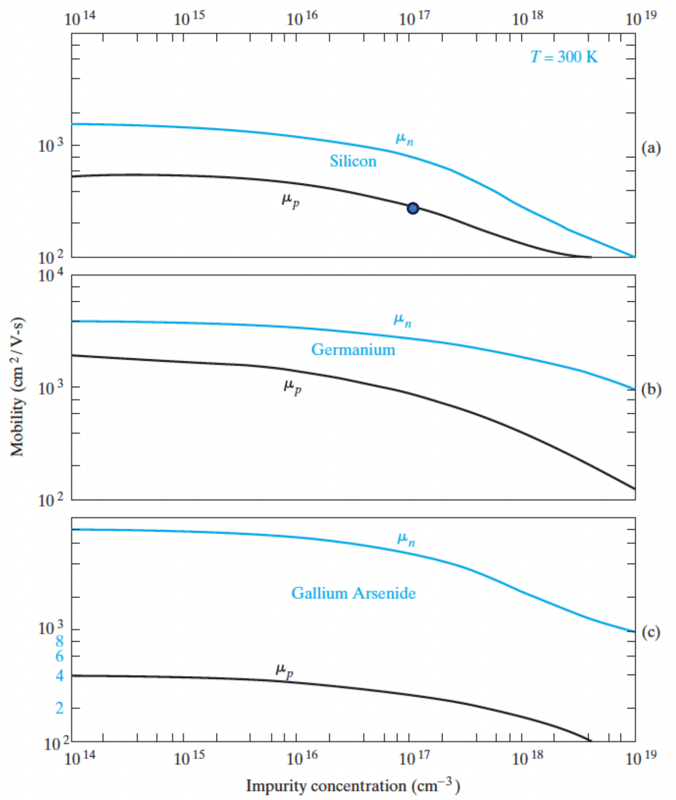


Figure 3-23 in textbook. Use for extrinsic semiconductors. Note you should use the total # impurities (Na+Nd)

Sources: Textbook

Recap: Diffusion (without recombination)

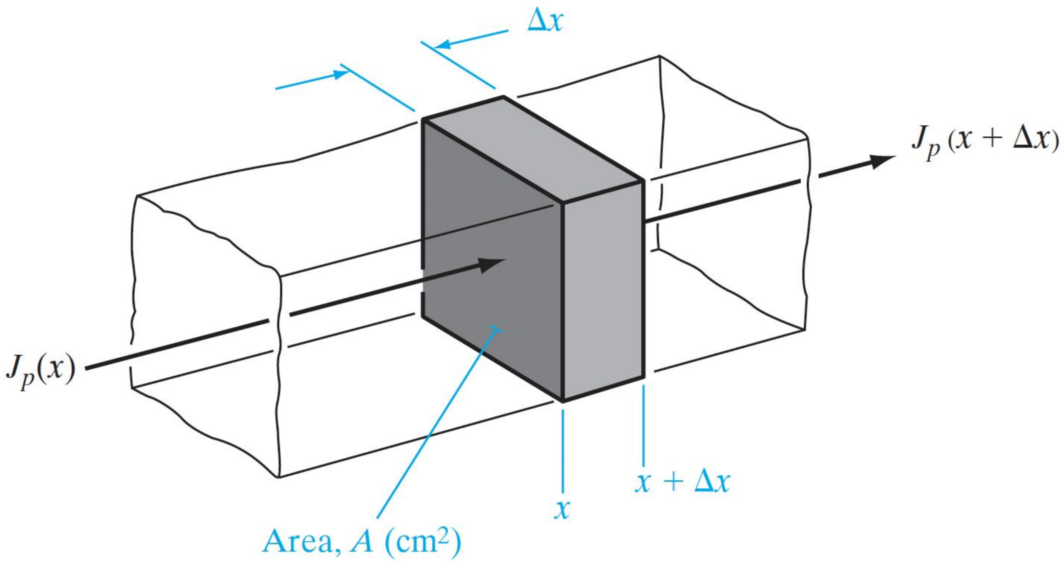
- Diffusion without recombination is driven by the ______________________
- We have the Einstein relation $\frac{D}{\mu} = \frac{kT}{q}$
- We can look up the mobility from plots, but make sure you use the total impurity concentration (___________)
- ______ at room temperature is ~0.026 V (you can memorize this, but be careful at temperatures different from ____)

- But what about in the case where we have recombination effects?
  - Recall: excess ______ can recombine

Sources: Textbook

Diffusion with Recombination

- Recombination can change carrier concentrations, so we must consider the effects on diffusion
- Consider an n-type semi sample with area ___ and a 'slice' of length ___
- Minority current density entering the area is:
- Minority current density leaving the area is:
- Simple counting
  - Rate of hole population increase = (current IN – current OUT) – hole recombination



Sources: Textbook

Diffusion with Recombination

- So let's count holes("bubbles"):
  - Recombination rate = # excess bubbles (δp) / recombination time (τ)
  - Current (#bubbles) IN – Current (#bubbles) OUT = $J_{IN} - J_{OUT}$ / dx
- What are the units?
  - Recombination rate:
  - Current:
- Therefore, we must account for width ___ (cm) of volume slice

$$\left.\frac{\partial p}{\partial t}\right|_{x \rightarrow x+\Delta x} = \frac{1}{q}\frac{J_p(x) - J_p(x+\Delta x)}{\Delta x} - \frac{\delta p}{\tau_p}$$

Rate of hole buildup = increase of hole concentration in $\delta x A$ per unit time − recombination rate

Sources: Textbook

Diffusion Length

- What does this mean in *steady-state*?
  - Distribution of excess carriers is maintained

- The diffusion equation in steady-state:

$$\boxed{\begin{aligned} \frac{d^2\delta n}{dx^2} &= \frac{\delta n}{D_n\tau_n} \equiv \frac{\delta n}{L_n^2} \\ \frac{d^2\delta p}{dx^2} &= \frac{\delta p}{D_p\tau_p} \equiv \frac{\delta p}{L_p^2} \end{aligned}}$$

- The diffusion length L_p = __________ is a figure of merit.
- $L_{p:}$ average length a carrier moves between ______________ and __________________

Sources: Textbook